Stosic   Homology of torus knots and links

$$\langle \times \rangle = \langle)(\rangle - q \langle \asymp \rangle$$

$D \qquad D_0 \qquad D_1 \qquad \langle \bigcirc \rangle = q + q^{-1}$

$J(D) = (-1)^{n_-} q^{n_+ - 2n_-} \langle D \rangle$ is a knot invariant

$$C(\times) := \underset{\text{cone}}{\mathcal{C}}\left(C()() \xrightarrow{f} C(\asymp) \right)$$

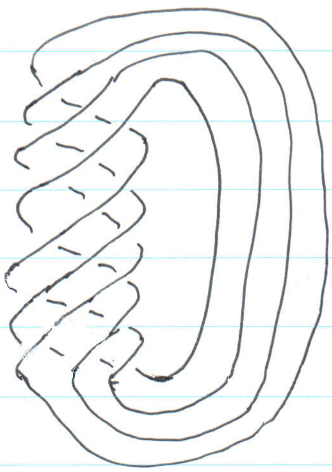
$\Rightarrow$ long exact sequence in KH

$$\to H^{i-1,j-1}(D_1) \to H^{i,j}(D) \to H^{i,j}(D_0) \to H^{i,j-1}(D_1) \to$$

$$\mathcal{H}^{i-n_-,\, j+n_+-2n_-}(D) = H^{i,j}(D)$$

torus knot

$T_{4,5}$

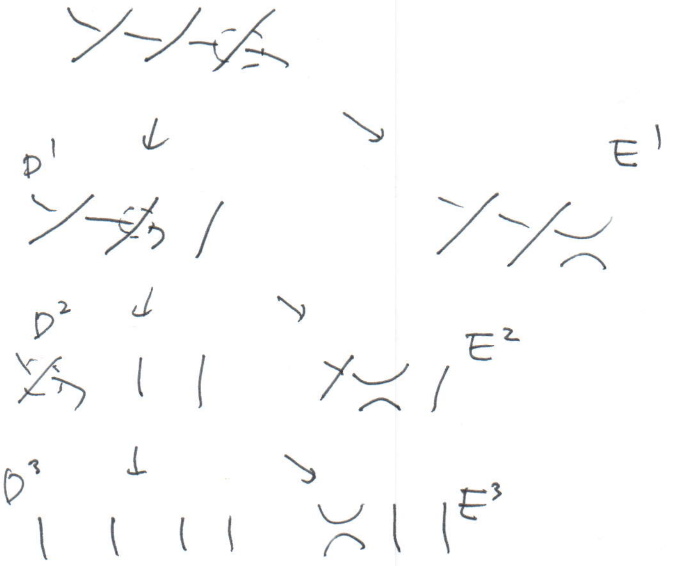


$T_{p,q} \sim T_{q,p}$

$D_{p,q}$

Th. $\quad H^{i,j}(D_{p,q}) \cong H^{i,j}(D_{p,q-1}) \qquad i < p+q-3$

$p < q \qquad \left(i < q-1+(p-2)\left[\frac{q}{p}\right] \right)$



$$\to \overset{0}{\overset{\|}{H^{i-1}(E^1)}} \to H^i(D) \to H^i(D^1) \to \overset{0}{\overset{\|}{H^i(E^1)}} \to$$

$$\to \overset{0}{\overset{\|}{H^{i-1}(E^2)}} \to H^i(D^1) \to H^i(D^2) \to \overset{0}{\overset{\|}{H^i(E^2)}} \to$$

$$\to \underset{0}{\underset{\|}{H^{i-1}(E^3)}} \to H^i(D^2) \to H^i(D^3) \to \underset{0}{\underset{\|}{H^i(E^3)}} \to$$

If $\quad H^i(E^j) = 0 \quad$ for $i < M$

$$\Rightarrow \quad H^i(D_{p,q}) \cong H^i(D_{p,q-1}) \quad i < M$$

$E^2 \sim P$ positive knot

$$\Rightarrow \mathcal{H}^i(P) = 0 \quad i < 0$$

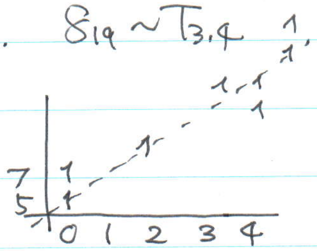
$$H^i(E^2) = 0 \quad i < n$$

$$n \geqq q - 1 + (p-2)\left[\frac{q}{p}\right]$$

Therefore we can take

$$M = q - 1 + (p-2)\left[\frac{q}{p}\right]$$

Cor 1. $8_{19} \sim T_{3,4}$



$$\mathcal{H}^{4,(p-1)(q-1)+5}(T_{p,2}) \cong \mathcal{H}^{4,11}(T_{3,4}) \qquad 3 \leq p < q$$

$\Rightarrow T_{p,2}$ are homologically thick.

Cor 2. existence of stable homology for torus knots

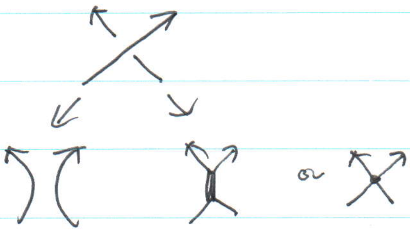
$$\lim_{q \to \infty} H^{i,j}(D_{p,q}) =: H^{i,j}(D_p)$$

$\because$ Take $q > i-p+3$ $\Rightarrow$ stabilize //

conjecturally HOMFLY version

Th. $H_n^{i,j}(D_{p,q}) \cong H_n^{i,j}(D_{p,q-1})$ $\quad i < p+q-3$

$\mathfrak{sl}(n)$

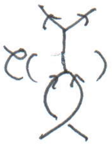


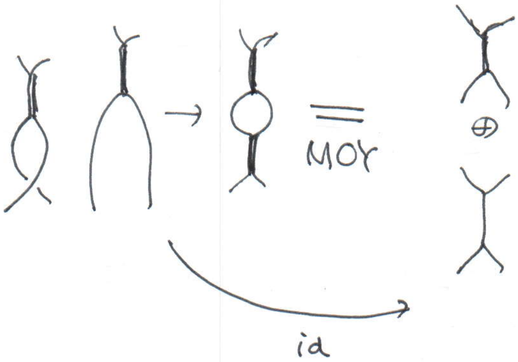
D_1 $\quad$ E_1

$$\to H_n^{i-1}(E_1) \to H_n^i(D) \to H_n^i(D_1) \to$$

E' を得る

(R2)'s analogy

$\mathcal{C}($  $) \cong \mathcal{C}($  $)[1]\{2\}$



$$C(\overline{E}_i) \cong C(X)[p+q-3]$$

positive

$$\Rightarrow H_n^i(\overline{E}_j) = 0 \qquad i < p+q-3$$

Cor. $\exists$ stable limit $\quad H_n^{i,j}(D_{p,q})$

computation $\quad H^i(T_{p,q}) = 0 \qquad i > \frac{pq}{2}$

$T_{2k,2kn}$

} n times

$(2k-1)2kn$ crossings

Th. $\quad H^i(T_{2k,2kn}) = 0 \qquad i > 2kn^2$

E.g. $\quad T_{4,4n} \qquad E^i \sim T_{2,2n-1}$ or $\bigcirc \qquad \mathcal{H}^i(T_{2,2n-2}) = 0$

$i \geq 2n$

$H^i(E_j) = 0 \qquad i > 8n$

$\Rightarrow \quad H^i(D_{4,4n}) \cong H^i(D_{4,4n-1}) \qquad i > 8n$

<u>Th.</u> $\mathcal{H}^{i,j}(T_{2k,2kn}) = 0 \qquad i > 2k^2 n$

or $j > 6k^2 n$

Th $\mathcal{H}^{i,j}(\overline{T}_{2k,2kn}) = 0$ if $i > 0$, $j > 0$

$\nearrow\nearrow \swarrow\swarrow$

in

半分の向きをかえる.

$\mathcal{H}^{i,j}(\overline{T}_{2k,2kn})$

$T_{4,4n}$

$$\begin{array}{c|l} \mathbb{Z}^2 & 0 \\ \mathbb{Z}^3 & -2 \\ \mathbb{Z} & -4 \end{array}$$

Th $\mathcal{H}^{0,-2i}(\overline{T}_{2k,2kn}) = \mathbb{Z}^{\binom{2k}{k-i} - \binom{2k}{k-i+1}}$

$\mathcal{H}^0$ total rk $\mathcal{H}^0 = \binom{2k}{k}$

$\mathbb{Z}^2 \quad 0$

$0 \to \mathbb{Z}^2 \to G \to \mathbb{Z} \to 0$

$\mathbb{Z} \quad -4$

Lee homology

$\Rightarrow$ rank $G \geqq 3$

<u>Conj.</u> $\lim_{n \to \infty} \mathcal{H}^{0,j}(\overline{T}_{2k,2kn}) = HH^i(H^k)$

$\vdots$

ring appearing in def.

Th. Conj. is true for $i=0$

$$HH^0(A) = Z(A) \quad \text{center}$$

Th (Khovanov)

$$Z(H^k) \qquad \overset{\text{basis}}{X_I} \qquad I \subset \{1, 2, \cdots, 2k\}$$

free

$$|I \cap \{1, \cdots, m\}| \leq \frac{m}{2}$$

$$\deg X_I = 2|I|$$

Cor $Z^{2j}(H^k) = HH^{0,2j}(H^k) = \mathbb{Z}^{\binom{2k}{j} - \binom{2k}{j-1}}$